

Sterile neutrinos: **implications for cosmology** **after BICEP2**

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Outline

- BICEP2 result
- Planck is in tension with BICEP2 and other astrophysical observations
- Sterile neutrino plays an important role in reducing tension between Planck and BICEP2
- Sterile neutrino could also relieve other tensions. --- Another cosmic concordance?
- Testing GR in the presence of sterile neutrino via measuring growth index using redshift-space distortions
- Neutrinos and dark energy after Planck and BICEP2

COSMOLOGY

Telescope captures view of gravitational waves

Images of the infant Universe reveal evidence for rapid inflation after the Big Bang.

20 MARCH 2014 | VOL 507 | NATURE | 281

NEWS IN FOCUS

BY RON COWEN IN CAMBRIDGE,
MASSACHUSETTS

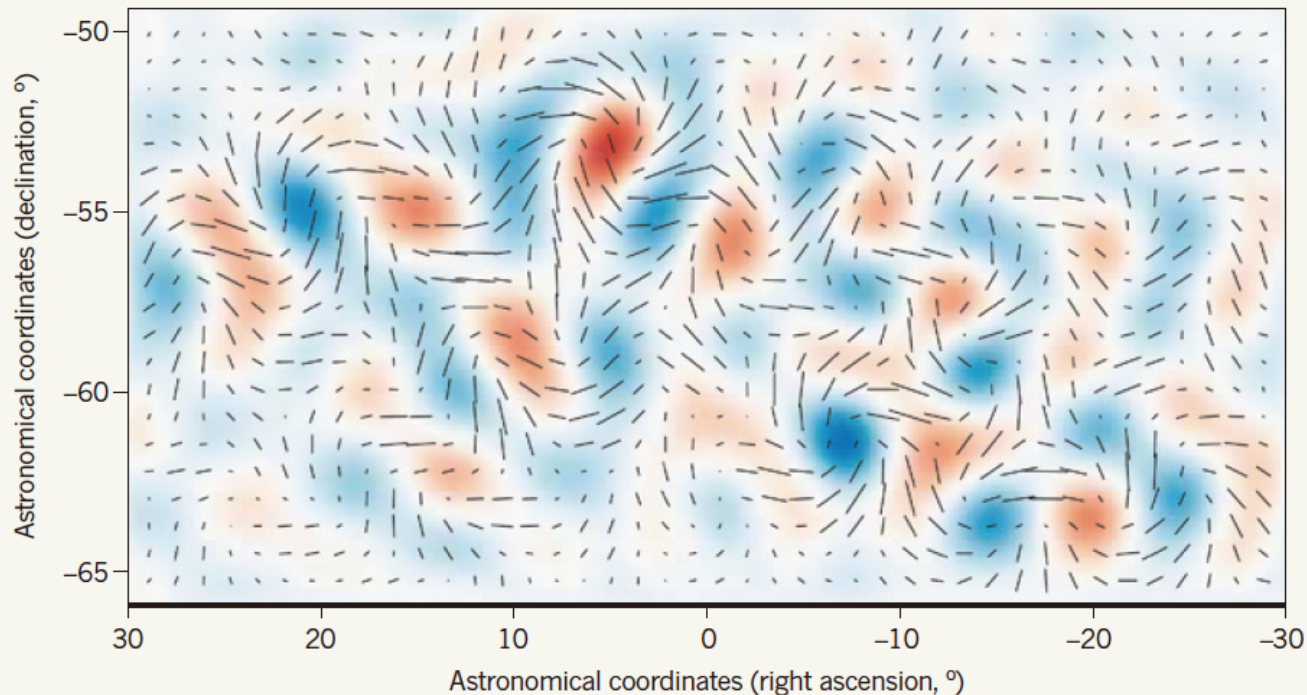
COSMIC CURL

The BICEP2 instrument observed a faint but distinctive twisting pattern, or spin, known as a curl or B-mode, in the polarization of the cosmic microwave background. This is the first evidence for gravitational waves generated by rapid inflation of the Universe some 13.8 billion years ago.

Spin intensity

■ Clockwise ■ Anti-clockwise

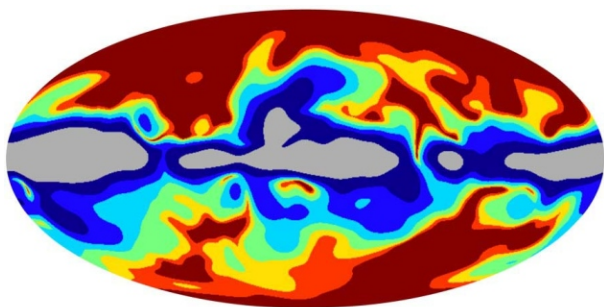
— Polarization strength and
orientation at different
spots on the sky.



COSMOLOGY

Big Bang finding challenged

Signal of gravitational waves was too weak to be significant, studies suggest.



Cross-checks of gravitational-wave results used this galactic-dust map from the Planck satellite.

NEWS | IN DEPTH

COSMOLOGY

Blockbuster claim could collapse in a cloud of dust

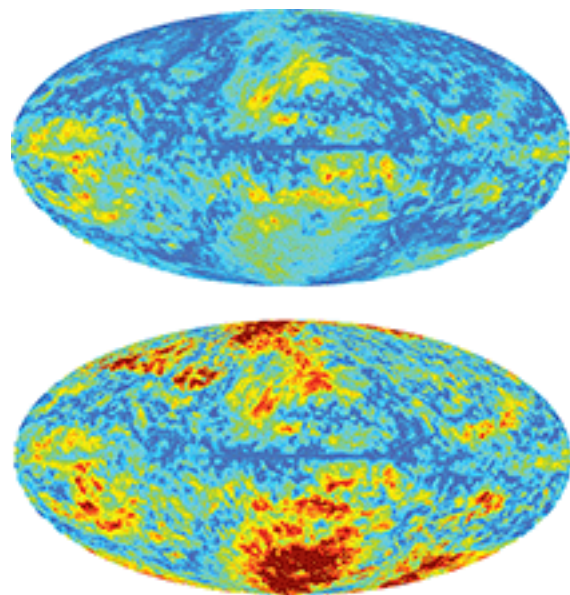
Smoking-gun evidence for cosmic inflation may actually be radiation from within our galaxy

sciencemag.org **SCIENCE**

23 MAY 2014 • VOL 344 ISSUE 6186

Mortonson, M. M. & Seljak, U. Preprint at <http://arxiv.org/abs/1405.5857> (2014).

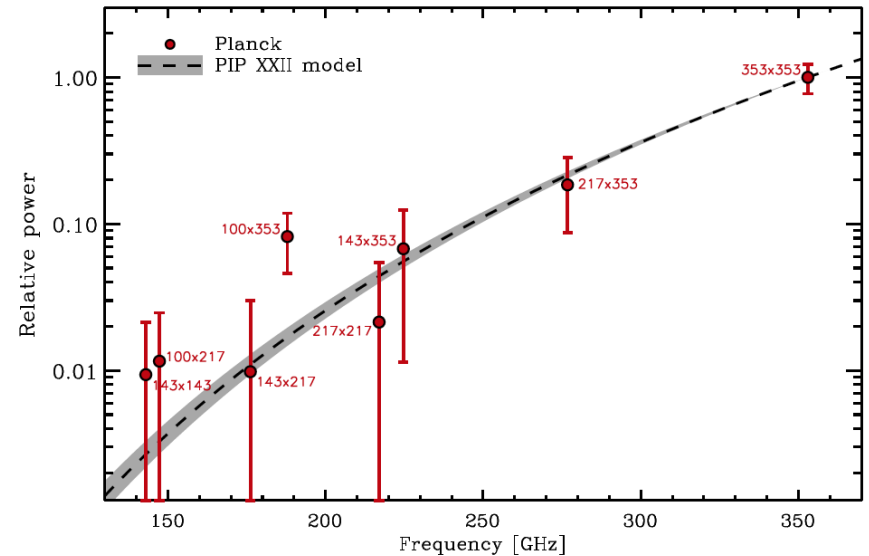
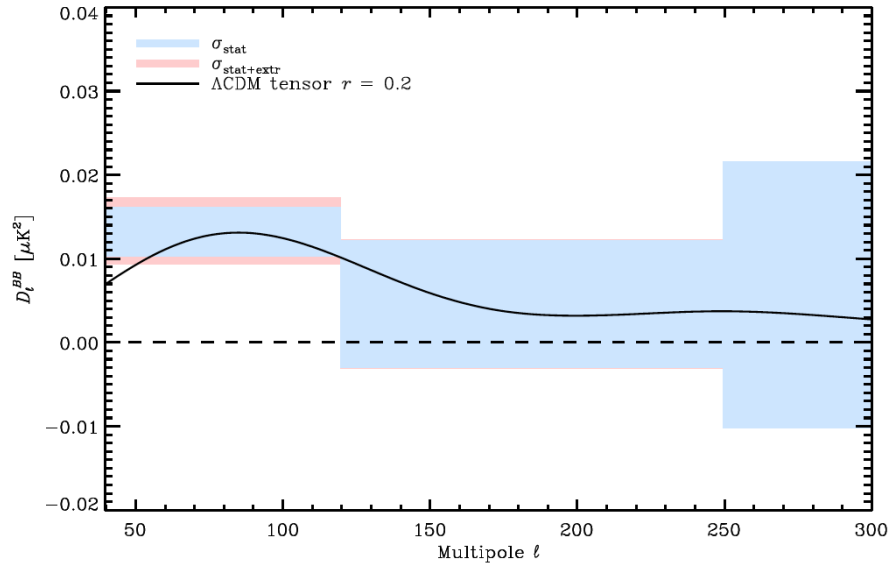
Flauger, R., Hill, J. C. & Spergel, D. N. Preprint at <http://arxiv.org/abs/1405.7351> (2014).



A reconstruction of the contaminated foreground map BICEP used (top) and the corrected map.

Planck intermediate results. XXX. **The angular power spectrum of polarized dust emission** **at intermediate and high Galactic latitudes**

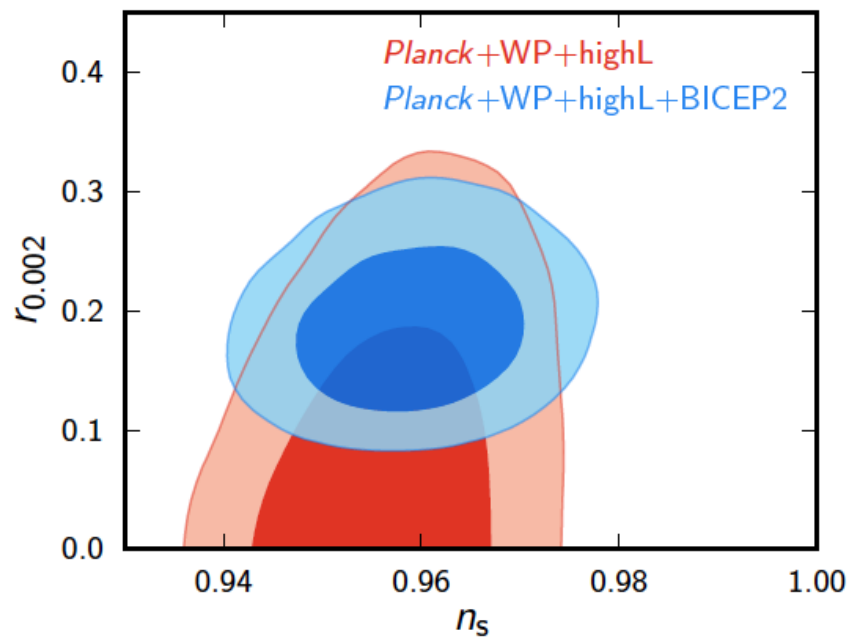
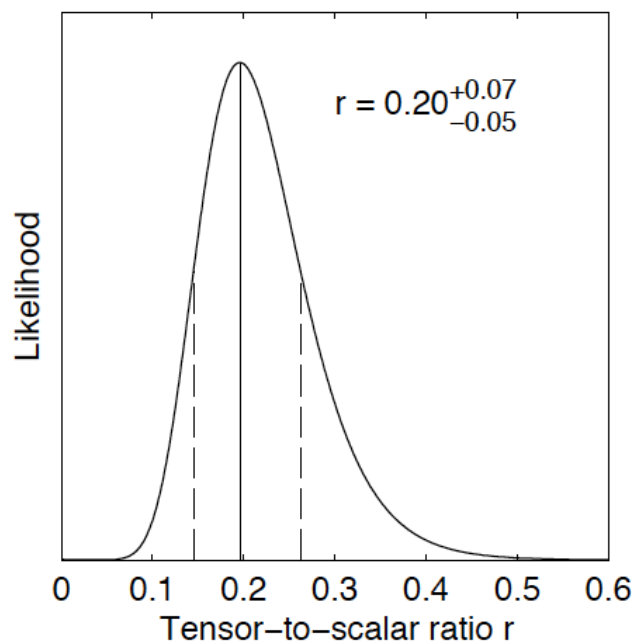
arXiv:1409.5738v1



- Planck dust polarization (353 GHz) alone could explain the BICEP2 result (from extrapolation to 150 GHz), but with large uncertainty
- There is no definitive conclusion
- Debates are still going on
- Look forward to the joint analysis of Planck and BICEP2 maps
- Before definitive conclusion, we cannot say that BICEP2 is wrong!

BICEP2: in tension with Planck

$$r = 0.20^{+0.07}_{-0.05}$$



Planck+WP+highL

$$r_{0.002} < 0.11 \quad (95\%; \text{no running}),$$

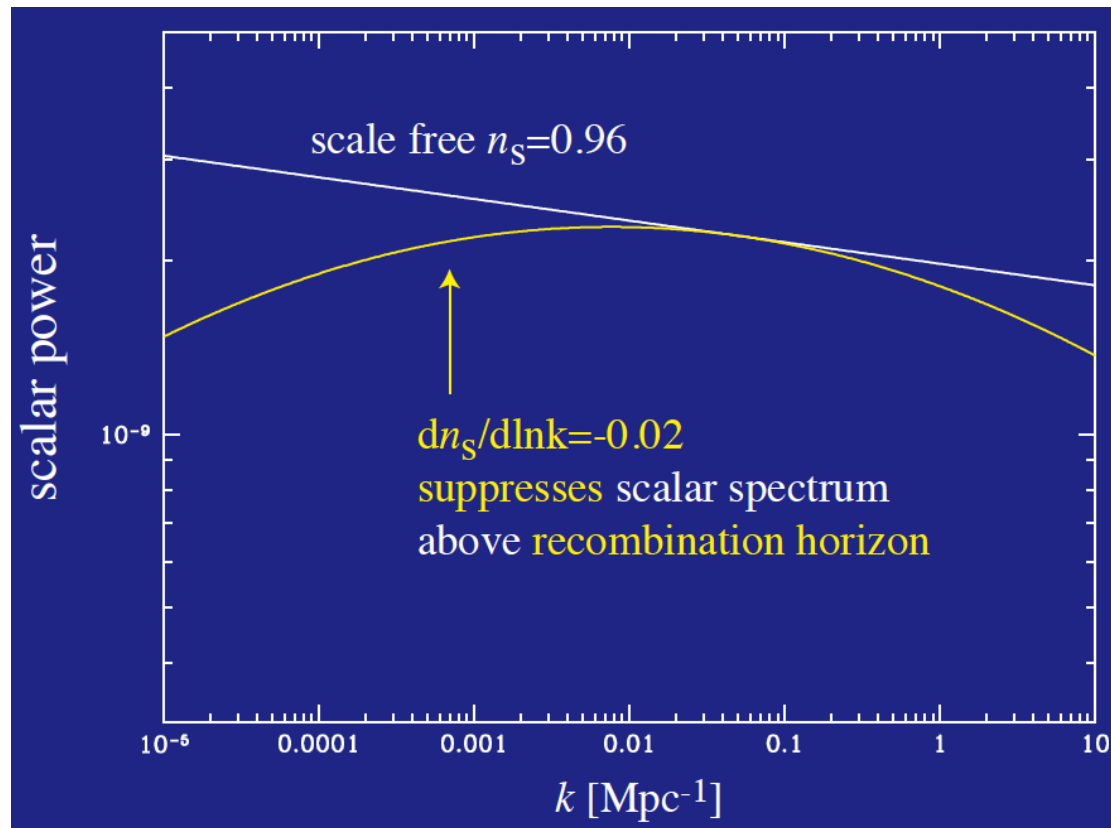
$$r_{0.002} < 0.26 \quad (95\%; \text{including running}).$$

$$dn_s/d \ln k = -0.022 \pm 0.010 \quad (68\%; \textit{Planck}+\textit{WP}+\textit{highL})$$

Large running is not good for inflation

$$dn_s/d\ln k = -0.022 \pm 0.010 \text{ (68\%; Planck+WP+highL)}$$

- Usual models: $dn_s/d\ln k \approx O(10^{-4})$
- Inflation models must be contrived

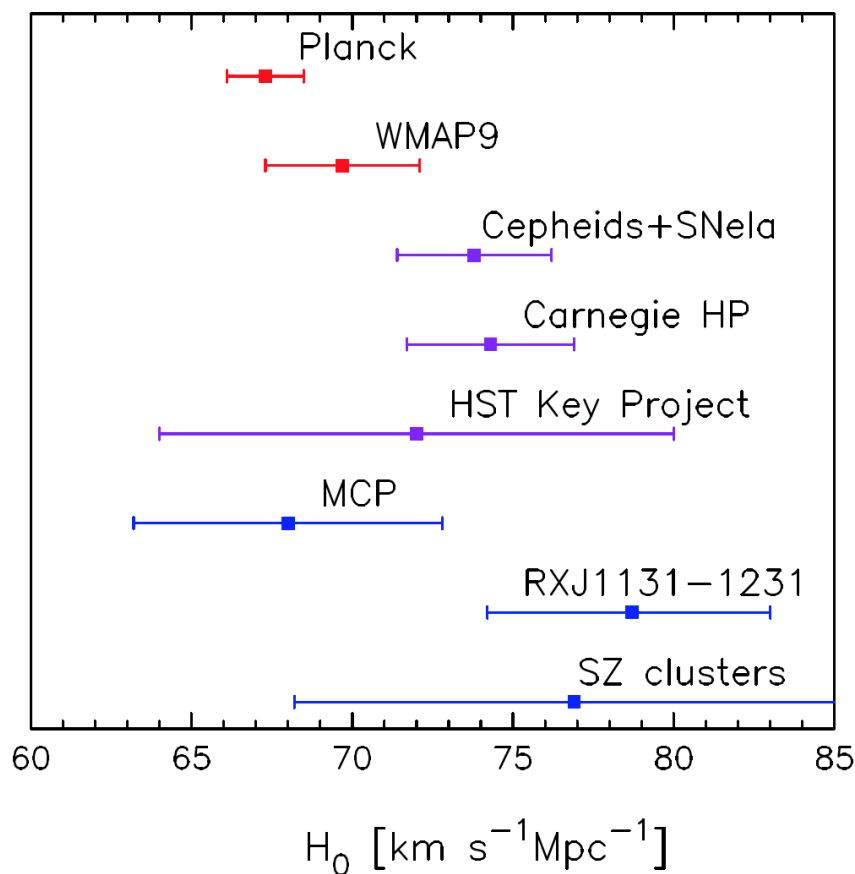


Planck: in tension with other astrophysical observations

$$H_0 = (67.3 \pm 1.2) \text{ km s}^{-1} \text{ Mpc}^{-1} \quad (68\%; \text{Planck}+\text{WP}+\text{highL})$$

$$H_0 = (73.8 \pm 2.4) \text{ km s}^{-1} \text{ Mpc}^{-1} \quad (\text{Cepheids}+\text{SNe Ia})$$

at about the 2.5σ level



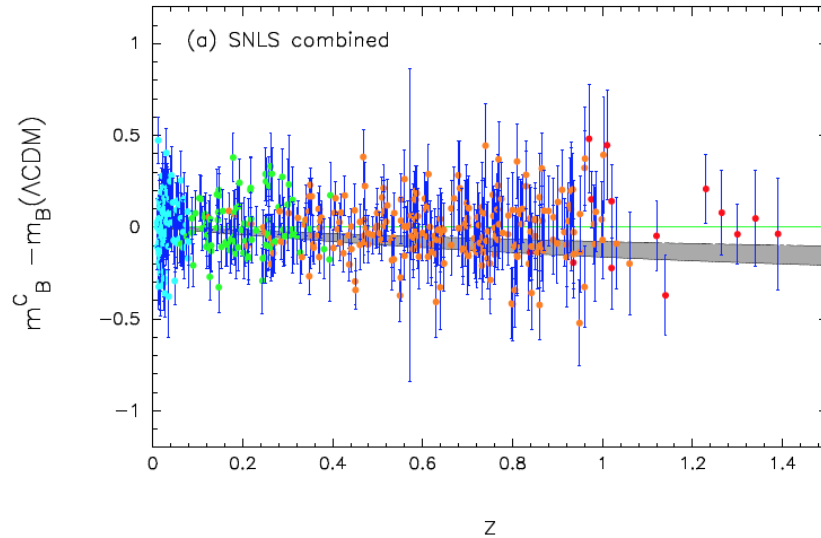
new physics?

Dark energy?

M. Li, X. -D. Li, Y. -Z. Ma, X. Zhang and Z. Zhang,
JCAP **1309**, 021 (2013).

Planck: in tension with other astrophysical observations

Type Ia supernovae



almost a 2σ discrepancy

may come from the new source of the systematic error, *e.g.*, the absence of evolution of the color-luminosity parameter β

S. Wang, Y. -H. Li and X. Zhang, "Exploring the evolution of color-luminosity parameter β and its effects on parameter estimation," *Phys. Rev. D* **89**, 063524 (2014) [arXiv:1310.6109 [astro-ph.CO]].

S. Wang, J. -J. Geng, Y. -L. Hu and X. Zhang, "Revisit of SNLS3 constraints on holographic dark energy: the effects of varying β and different light-curve fitters," arXiv:1312.0184 [astro-ph.CO].

Cosmic shear

$$\sigma_8 (\Omega_m/0.27)^{0.46} = 0.89 \pm 0.03 \quad (68\%; \text{Planck+WP+highL})$$

cosmic shear data of the weak lensing from the CFHTLenS survey

$$\sigma_8 (\Omega_m/0.27)^{0.46} = 0.774 \pm 0.040$$

at about the 2σ level

Counts of rich clusters

$$\sigma_8 (\Omega_m/0.27)^{0.3} = 0.87 \pm 0.02 \quad (68\%; \text{Planck+WP+highL})$$

Planck thermal Sunyaev-Zeldovich (SZ) clusters

$$\sigma_8 (\Omega_m/0.27)^{0.3} = 0.78 \pm 0.01$$

significant (around 3σ) discrepancy

Sterile neutrinos help reconcile the observational results of primordial gravitational waves from Planck and BICEP2

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We show that involving a sterile neutrino species in the Λ CDM+ r model can help relieve the tension about the tensor-to-scalar r between the Planck temperature data and the BICEP2 B-mode polarization data. Such a model is called the Λ CDM+ $r+\nu_s$ model in this paper. Comparing to the Λ CDM+ r model, there are two extra parameters, N_{eff} and $m_{\nu_s}^{\text{eff}}$, in the Λ CDM+ $r+\nu_s$ model. We show that in this model the tension between Planck and BICEP2 can be greatly relieved at the cost of the increase of n_s . However, comparing with the Λ CDM+ $r+dn_s/d\ln k$ model that can significantly reduce the tension between Planck and BICEP2 but also makes trouble to inflation due to the large running of the spectral index of order 10^{-2} produced, the Λ CDM+ $r+\nu_s$ model is much better for inflation. By including a sterile neutrino species in the standard cosmology, besides the tension with BICEP2, the other tensions of Planck with other astrophysical data, such as the H_0 direct measurement, the Sunyaev-Zeldovich cluster counts, and the galaxy shear data, can all be significantly relieved. So, this model seems to be an economical choice. Combining the Planck temperature data, the WMAP-9 polarization data, and the baryon acoustic oscillation data with all these astrophysical data (including BICEP2), we find that in the Λ CDM+ $r+\nu_s$ model $n_s = 0.999^{+0.012}_{-0.011}$, $r = 0.21^{+0.04}_{-0.05}$, $N_{\text{eff}} = 3.961^{+0.318}_{-0.325}$ and $m_{\nu_s}^{\text{eff}} = 0.511^{+0.120}_{-0.133}$ eV. Thus, our results prefer $\Delta N_{\text{eff}} > 0$ at the 2.8σ level and a nonzero mass of sterile neutrino at the 4.2σ level.

arXiv:1403.7028v2 [astro-ph.CO]

27 Mar 2014

Neutrinos help reconcile Planck measurements with both Early and Local Universe

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In light of the recent BICEP2 B-mode polarization detection, which implies a large inflationary tensor-to-scalar ratio $r = 0.2^{+0.07}_{-0.08}$, we re-examine the evidence for an extra sterile massive neutrino, originally invoked to account for the tension between the cosmic microwave background (CMB) temperature power spectrum and local measurements of the expansion rate H_0 and cosmological structure. With only the standard active neutrinos and power-law scalar spectra, this detection is in tension with the upper limit of $r < 0.11$ (95% confidence) from the lack of a corresponding low multipole excess in the temperature anisotropy from gravitational waves. An extra sterile species with the same energy density as is needed to reconcile the CMB data with H_0 measurements can also alleviate this new tension. By combining data from the Planck and ACT/SPT temperature spectra, WMAP9 polarization, H_0 , baryon acoustic oscillation and local cluster abundance measurements with BICEP2 data, we find the joint evidence for a sterile massive neutrino increases to $\Delta N_{\text{eff}} = 0.81 \pm 0.25$ for the effective number and $m_s = 0.47 \pm 0.13 \text{ eV}$ for the effective mass or 3.2σ and 3.6σ evidence respectively.

arXiv:1403.8049v1

31 Mar 2014

During completion of this work a similar study appeared [33]; our addition of the SPT/ACT data sets place stronger upper limits on the allowed ΔN_{eff} .

Sterile neutrinos

Δm^2 of about $7 \times 10^{-5} \text{ eV}^2$

Sun, long-baseline reactor

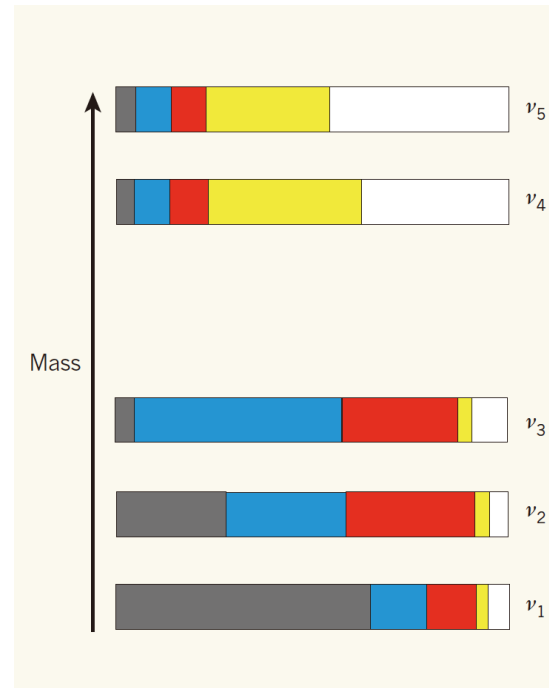
Δm^2 of around $2 \times 10^{-3} \text{ eV}^2$

atmosphere, long-baseline accelerator

Δm^2 of about 1 eV^2

short-baseline accelerator, reactor

Anomaly! Sterile?



- Sterile: they interact only by gravity, and not by weak interaction (would not affect the width of Z^0)
- Through their mixing with active neutrinos and their interactions with gravity, sterile neutrinos could have a big effect on astrophysics and cosmology
- Detection: search for oscillations between active and sterile neutrinos
- Cosmological evidence: through their gravitational effects on galaxy formation and the evolution of the universe

Λ CDM+ r + ν_s model

The Λ CDM+ r + $N_{\text{eff}}+m_{\nu,\text{sterile}}^{\text{eff}}$ model

Consider the massive sterile neutrino. In this case, the total mass of active neutrinos $\sum m_\nu$ is kept fixed at 0.06 eV, but we add one massive sterile neutrino in the model. Thus, two additional parameters, N_{eff} and $m_{\nu,\text{sterile}}^{\text{eff}}$, are introduced.

- Planck+WP: the CMB TT angular power spectrum data from Planck [2], in combination with the large-scale EE and TE polarization power spectrum data from 9-year WMAP [21].
- BAO: the latest measurement of the cosmic distance scale from the Data Release 11 (DR11) galaxy sample of the Baryon Oscillation Spectroscopic Survey (BOSS) [that is part of the Sloan Digital Sky Survey III (SDSS-III)]: $D_V(0.32)(r_{d,\text{fid}}/r_d) = (1264 \pm 25)$ Mpc and $D_V(0.57)(r_{d,\text{fid}}/r_d) = (2056 \pm 20)$ Mpc, with $r_{d,\text{fid}} = 149.28$ Mpc [22].
- H_0 : the direct measurement of the Hubble constant using the cosmic distance ladder in the Hubble Space Telescope observations of Cepheid variables and type Ia supernovae, $H_0 = (73.8 \pm 2.4)$ km s⁻¹ Mpc⁻¹ [23].
- SZ: the counts of rich clusters of galaxies from the sample of Planck thermal Sunyaev-Zeldovich (SZ) clusters constrain the combination of σ_8 and Ω_m , $\sigma_8(\Omega_m/0.27)^{0.3} = 0.78 \pm 0.01$ [24].
- Lensing: the CMB lensing power spectrum $C_\ell^{\phi\phi}$ from Planck [25], and also the combination of σ_8 and Ω_m given by the cosmic shear data of the weak lensing from the CFHTLenS survey, $\sigma_8(\Omega_m/0.27)^{0.46} = 0.774 \pm 0.040$ [26].
- BICEP2: the CMB angular power spectra (TT, TE, EE, and BB) data from BICEP2 [1].

Sterile neutrino: A concordance model

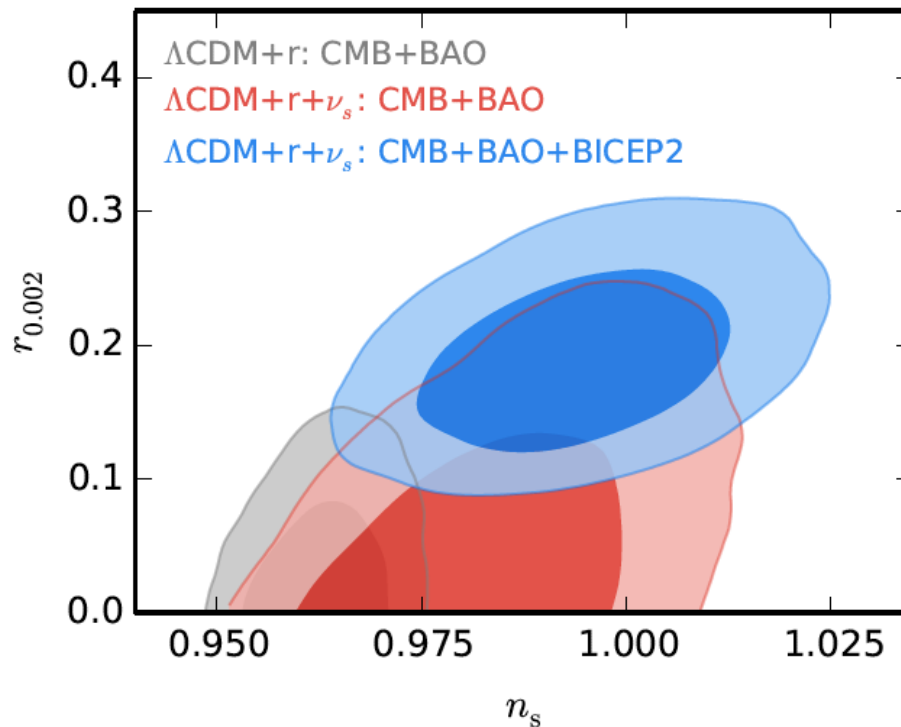
Using only Planck+WP+BAO can lead to $r_{0.002} < 0.20$ (95% CL)

including H_0 +SZ+Lensing can give $r_{0.002} < 0.23$ (95% CL)

consistent with the BICEP2

CMB+BAO+BICEP

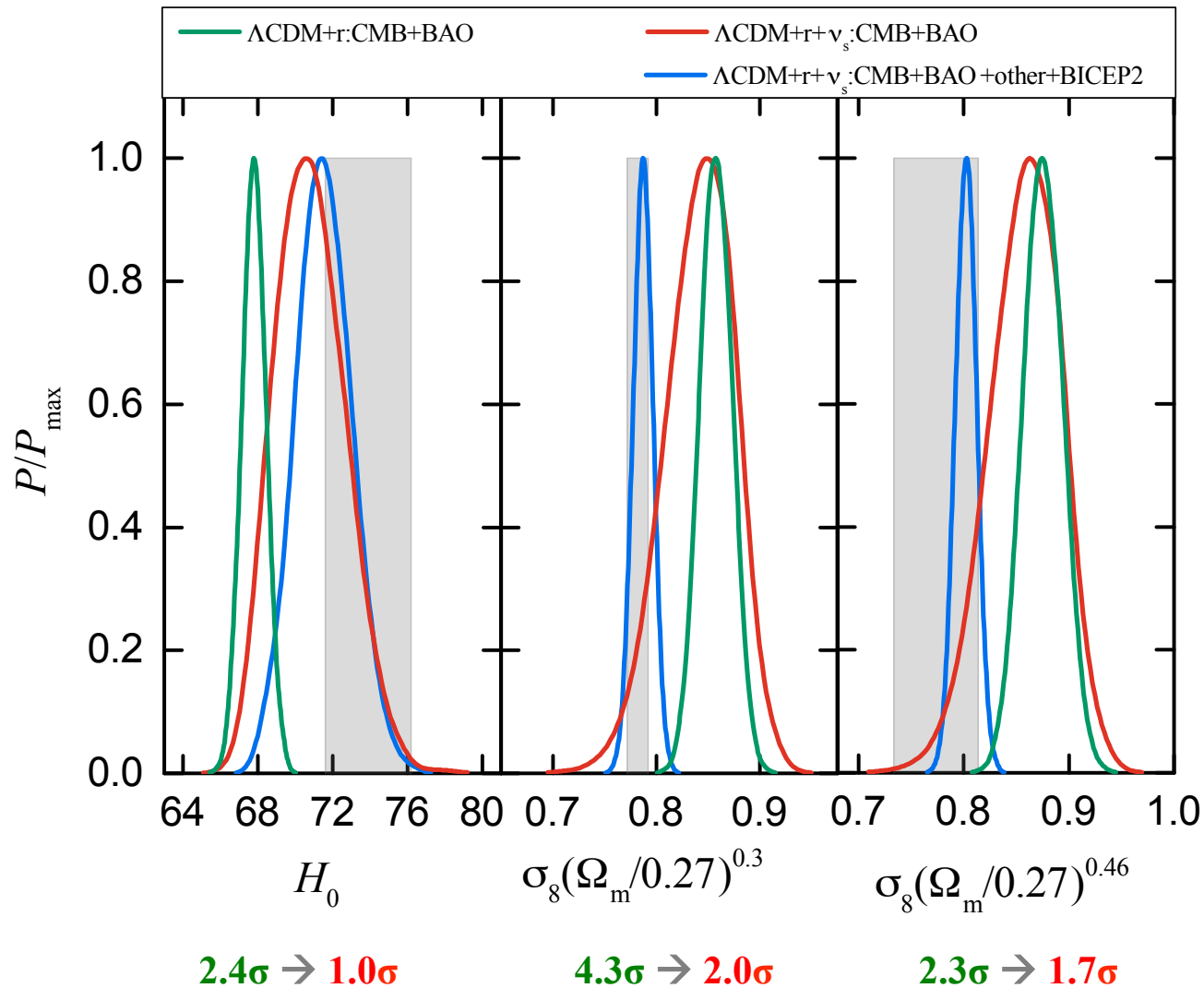
$$n_s = 0.994^{+0.012}_{-0.013} \text{ and } r = 0.19^{+0.04}_{-0.05} \text{ (95\% CL)}$$



$$\left. \begin{array}{l} N_{\text{eff}} = 3.72^{+0.32}_{-0.40} \text{ (68\% CL)} \\ m_{\nu, \text{sterile}}^{\text{eff}} < 0.51 \text{ eV (95\% CL)} \end{array} \right\} \text{ (Planck+WP+BAO)}$$

$$\Delta N_{\text{eff}} > 0 \\ 1.7\sigma$$

Sterile neutrino: A concordance model



Sterile neutrino: A concordance model

$$n_s = 0.999^{+0.012}_{-0.011}, \quad r = 0.21^{+0.04}_{-0.05}$$

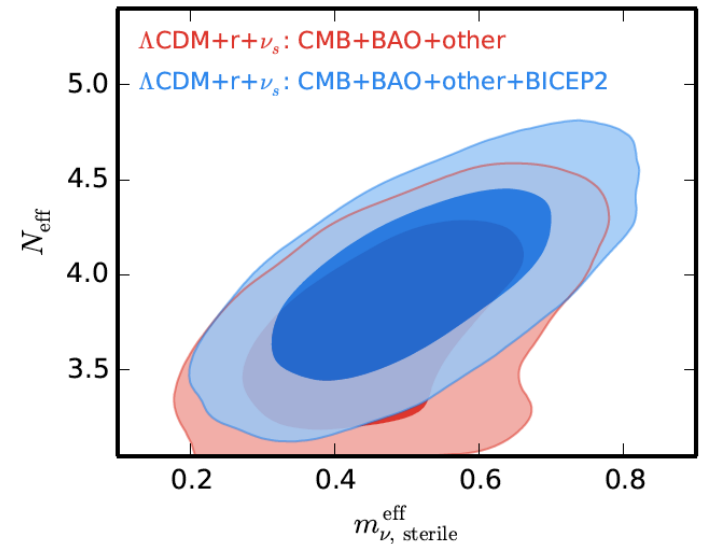
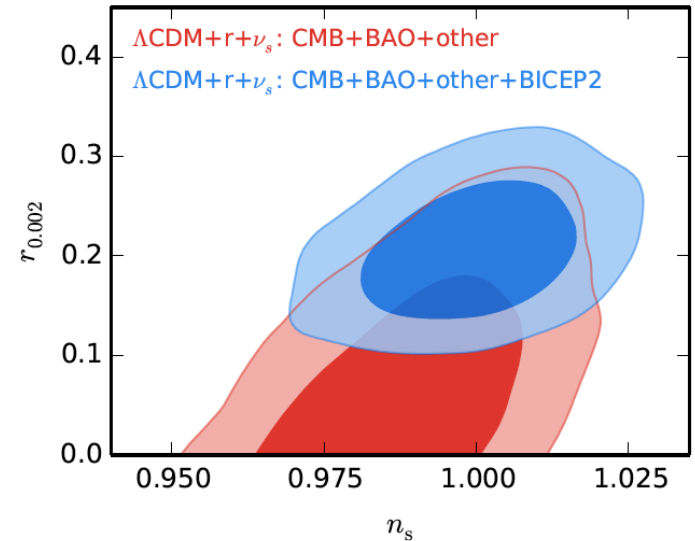
(68% CL; Planck+WP+BAO+ H_0 +SZ+Lensing+BICEP2)

$$\left. \begin{array}{l} N_{\text{eff}} = 3.75^{+0.34}_{-0.37} \\ m_{\nu, \text{sterile}}^{\text{eff}} = 0.48^{+0.11}_{-0.13} \text{ eV} \end{array} \right\} \begin{array}{l} 1.9\sigma \\ 3.6\sigma \end{array}$$

(68% CL; Planck+WP+BAO+ H_0 +SZ+Lensing)

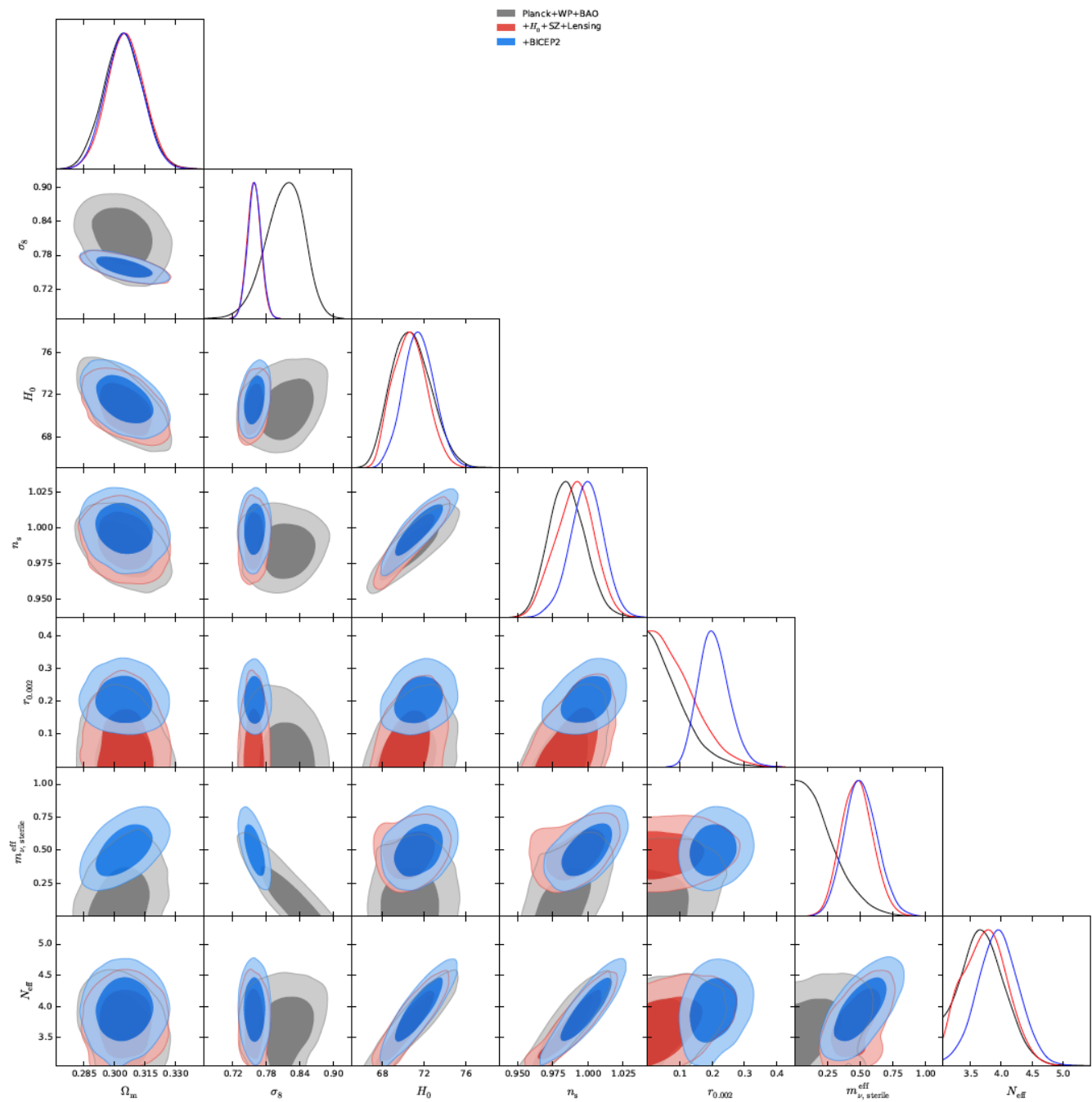
$$\left. \begin{array}{l} N_{\text{eff}} = 3.95 \pm 0.33 \\ m_{\nu, \text{sterile}}^{\text{eff}} = 0.51^{+0.12}_{-0.13} \text{ eV} \end{array} \right\} \begin{array}{l} 2.7\sigma \\ 3.9\sigma \end{array}$$

(68% CL; Planck+WP+BAO+ H_0 +SZ+Lensing+BICEP2)



The best-fit results, $\Delta N_{\text{eff}} \approx 1$ and $m_{\text{sterile}}^{\text{thermal}} \approx m_{\nu, \text{sterile}}^{\text{eff}} \approx 0.5 \text{ eV}$

- The results indicate a fully thermalized sterile neutrino with sub-eV mass.
- However, the short baseline neutrino oscillation experiments prefer the mass of sterile neutrino at around 1 eV.
- So, there is still a tension on the sterile neutrino mass between the cosmological data and the short-baseline neutrino oscillation data.
- The implication of this tension for cosmology deserves further investigations.



Parameters	Planck+WP+BAO		+ H_0 +SZ+Lensing		+BICEP2	
	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits
$\Omega_b h^2$	0.02229	0.0225 ± 0.0003	0.02261	$0.02277^{+0.00027}_{-0.00028}$	0.02287	0.02282 ± 0.00028
$\Omega_c h^2$	0.1257	$0.1273^{+0.0054}_{-0.0061}$	0.1168	$0.1241^{+0.0052}_{-0.0056}$	0.1256	$0.1271^{+0.0049}_{-0.0048}$
$100\theta_{\text{MC}}$	1.04047	$1.0405^{+0.00076}_{-0.00075}$	1.04162	1.04078 ± 0.00074	1.0411	1.0405 ± 0.0007
τ	0.088	$0.097^{+0.014}_{-0.015}$	0.101	$0.106^{+0.014}_{-0.016}$	0.113	$0.107^{+0.014}_{-0.016}$
$m_{\nu, \text{sterile}}^{\text{eff}}$	0.00	< 0.51	0.38	$0.48^{+0.11}_{-0.13}$	0.51	$0.51^{+0.12}_{-0.13}$
N_{eff}	3.51	$3.72^{+0.32}_{-0.40}$	3.28	$3.75^{+0.34}_{-0.37}$	3.88	3.95 ± 0.33
n_s	0.974	$0.985^{+0.012}_{-0.014}$	0.976	$0.991^{+0.015}_{-0.013}$	0.998	0.999 ± 0.011
$\ln(10^{10} A_s)$	3.1	$3.12^{+0.030}_{-0.034}$	3.107	$3.131^{+0.031}_{-0.035}$	3.147	$3.14^{+0.031}_{-0.035}$
$r_{0.05}$	0.00	< 0.19	0.00	< 0.21	0.173	$0.191^{+0.036}_{-0.041}$
Ω_Λ	0.6998	0.6956 ± 0.0093	0.6984	$0.6944^{+0.0087}_{-0.0088}$	0.6975	$0.6952^{+0.0088}_{-0.0087}$
Ω_m	0.3002	0.3044 ± 0.0093	0.3016	$0.3056^{+0.0088}_{-0.0087}$	0.3025	$0.3048^{+0.0087}_{-0.0088}$
σ_8	0.839	$0.812^{+0.038}_{-0.029}$	0.758	$0.758^{+0.011}_{-0.012}$	0.756	0.759 ± 0.012
H_0	70.4	$70.8^{+1.7}_{-2.1}$	69.1	$70.7^{+1.5}_{-1.8}$	71.5	$71.5^{+1.4}_{-1.6}$
$r_{0.002}$	0.00	< 0.20	0.00	< 0.23	0.184	$0.207^{+0.041}_{-0.052}$
$-\ln \mathcal{L}_{\text{max}}$	4904.07		4913.24		4933.82	

Table 4. Fitting results for the $\Lambda\text{CDM}+r+N_{\text{eff}}+m_{\nu, \text{sterile}}^{\text{eff}}$ model. We quote $\pm 1\sigma$ errors, but for the parameters that cannot be well constrained, we quote the 95% CL upper limits.

Measuring growth index in a universe with sterile neutrinos

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Consistency tests for the general relativity (GR) can be performed by constraining the growth index γ using the measurements of redshift-space distortions (RSD) in conjunction with other observations. In previous studies, deviations from the GR expected value of $\gamma \approx 0.55$ at the $2\text{--}3\sigma$ level were found. In this work, we reconsider the measurement of γ in a universe with sterile neutrinos. We constrain the sterile neutrino cosmological model using the RSD measurements combined with the cosmic microwave background data (Planck temperature data plus WMAP 9-yr polarization data), the baryon acoustic oscillation data, the Hubble constant direct measurement, the Planck Sunyaev-Zeldovich cluster counts data, and the galaxy shear data. We obtain the constraint result of the growth index, $\gamma = 0.584^{+0.047}_{-0.048}$, well consistent with the GR expected value (the consistency is at the 0.6σ level). For the parameters of sterile neutrino, we obtain $N_{\text{eff}} = 3.62^{+0.26}_{-0.42}$ and $m_{\nu, \text{sterile}}^{\text{eff}} = 0.48^{+0.11}_{-0.14}$ eV. We also consider the BICEP2 data and perform an analysis on the model with tensor modes. Similar fit results are obtained, showing that once light sterile neutrino is considered in the universe, GR will become well consistent with the current observations.

arXiv:1408.4603v1

Since dark energy and MG theories can in principle predict identical expansion histories, a potential way of distinguishing between them is to probe and compare the different structure growth histories of them.

linear growth rate $f(a) = d \ln D(a) / d \ln a$

growth index

$$f(a) = \Omega_m(a)^\gamma \quad \gamma \approx 0.55 \quad \text{for GR}$$

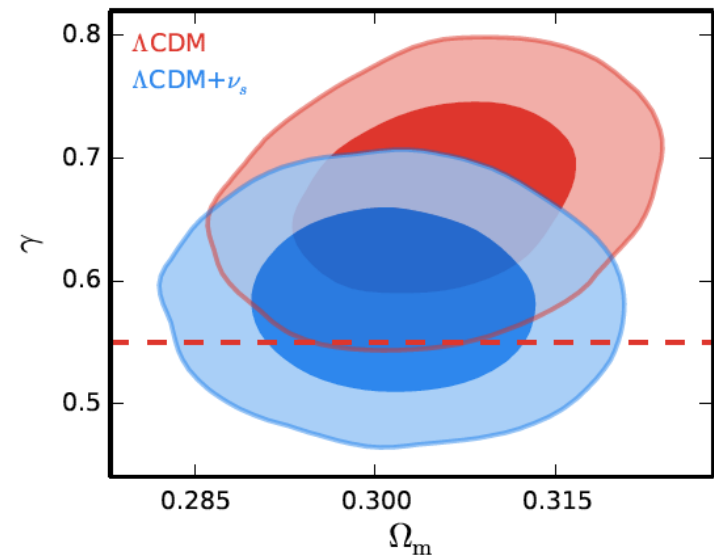
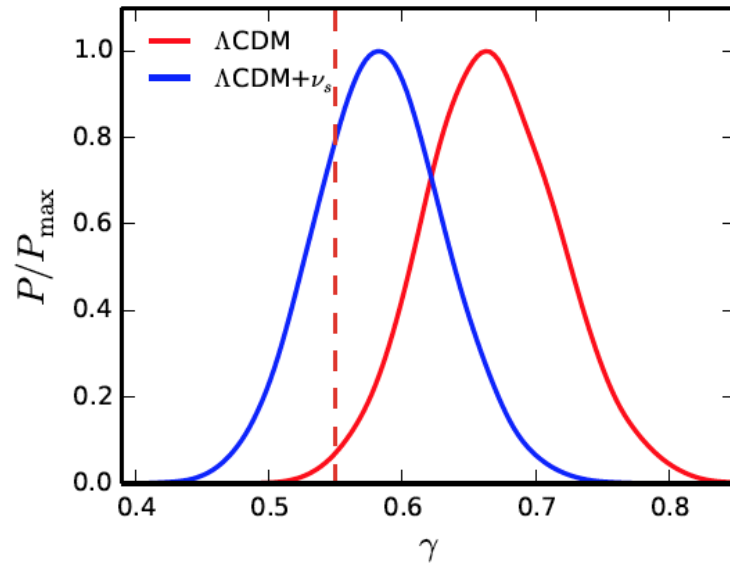
Wang & Steinhardt 1998
Linder 2005

Red-space distortions (RSD) measure $f(z)$; in practice

$$f(a)\sigma_8(a) = d\sigma_8(a)/d \ln a$$

- RSD constrain γ : testing GR
- Samushia et al. 2012 (BOSS-DR9): $\gamma = 0.75 \pm 0.09$
- Beutler et al. 2013 (BOSS-DR11): $\gamma = 0.772^{+0.124}_{-0.097}$
- Discrepant from GR at 2-3 σ level

Constraining growth index from RSD



$$\gamma = 0.584^{+0.047}_{-0.048}$$

the consistency with GR is at the 0.6σ level

Neutrinos and dark energy after Planck and BICEP2: data consistency tests and cosmological parameter constraints

arXiv:1408.0481v1

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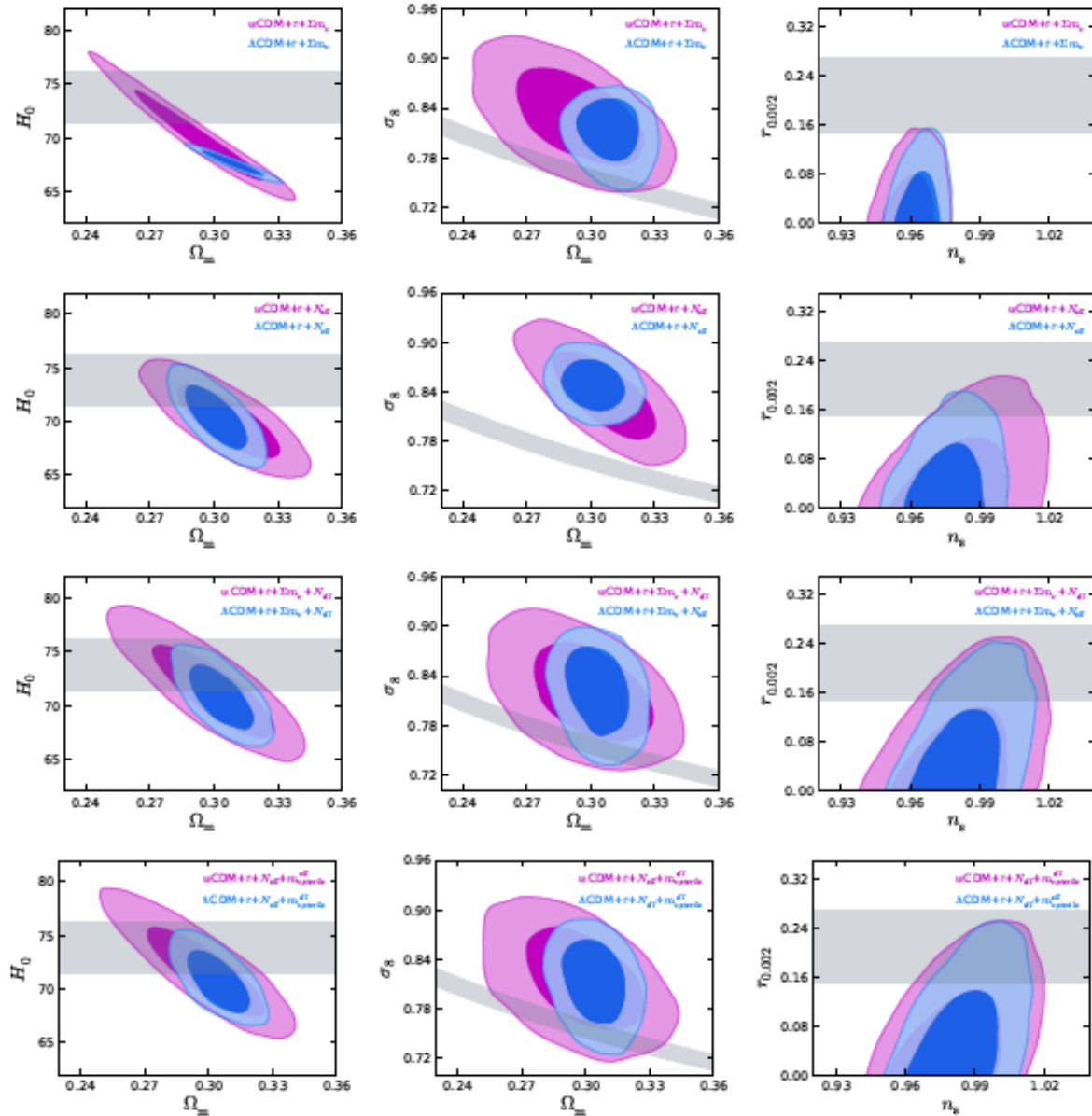
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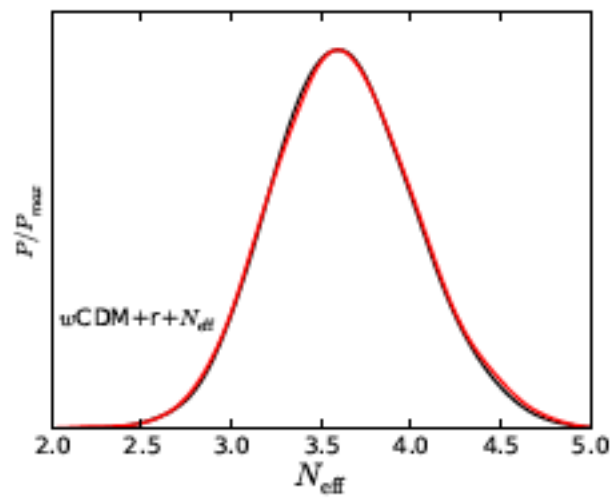
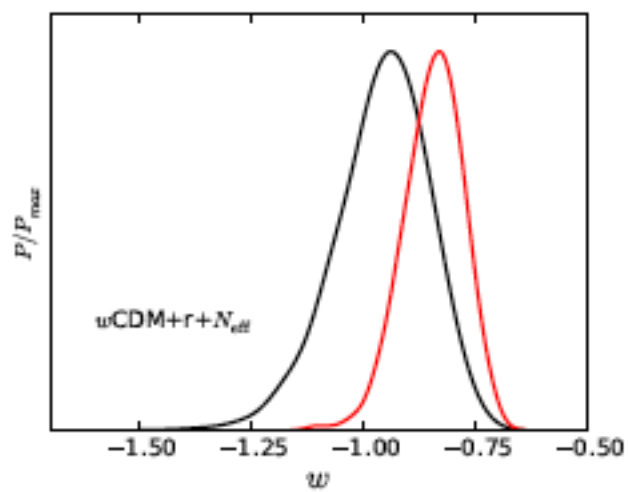
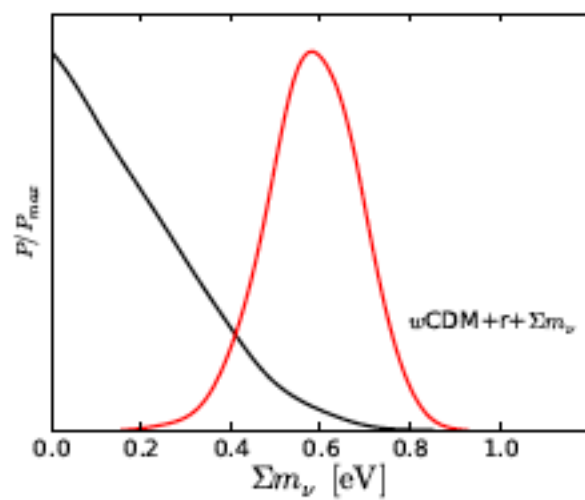
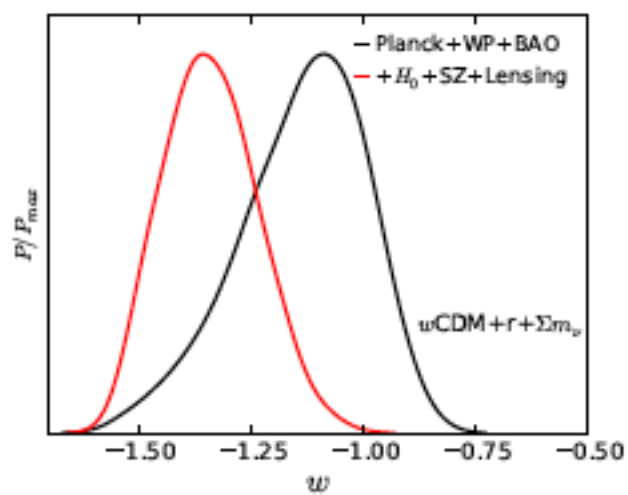
Abstract. The detection of the B-mode polarization of the cosmic microwave background (CMB) by the BICEP2 experiment implies that the tensor-to-scalar ratio r should be involved in the base standard cosmology. In this paper, we extend the Λ CDM+ r +neutrino/dark radiation models by replacing the cosmological constant with the dynamical dark energy with constant w . Four neutrino plus dark energy models are considered, i.e., the w CDM+ r + $\sum m_\nu$, w CDM+ r + N_{eff} , w CDM+ r + $\sum m_\nu$ + N_{eff} , and w CDM+ r + N_{eff} + $m_{\nu,\text{sterile}}^{\text{eff}}$ models. The current observational data considered in this paper include the Planck temperature data, the WMAP 9-year polarization data, the baryon acoustic oscillation data, the Hubble constant direct measurement data, the Planck Sunyaev-Zeldovich cluster counts data, the Planck CMB lensing data, the cosmic shear data, and the BICEP2 polarization data. We test the data consistency in the four cosmological models, and then combine the consistent data sets to perform joint constraints on the models. We focus on the constraints on the parameters w , $\sum m_\nu$, N_{eff} , and $m_{\nu,\text{sterile}}^{\text{eff}}$.

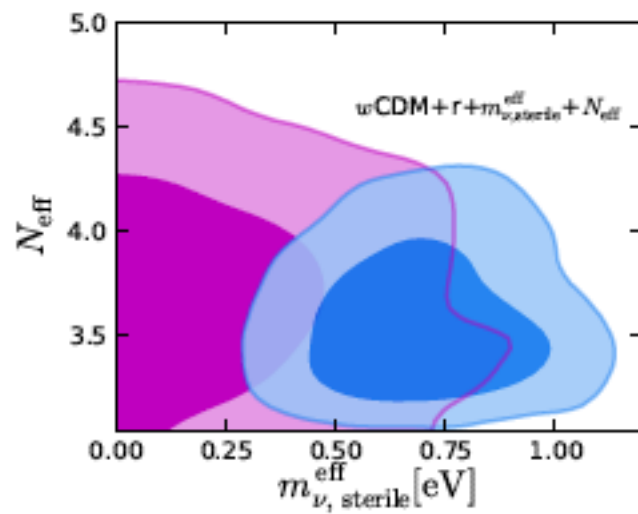
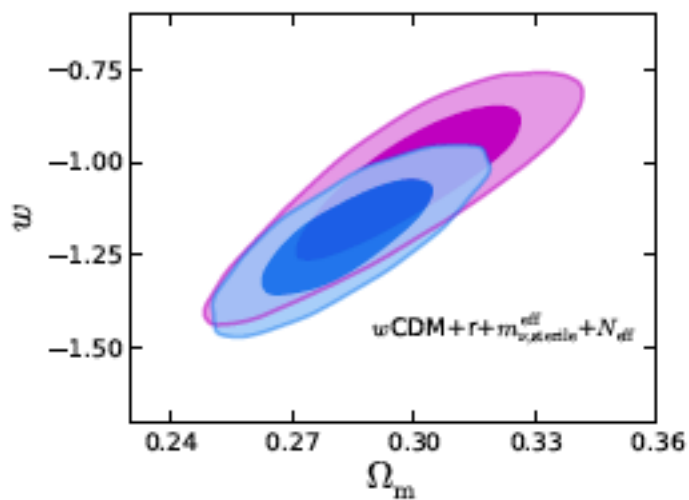
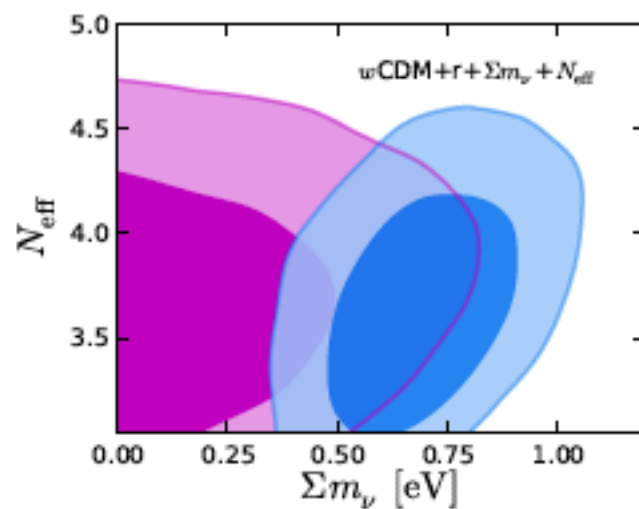
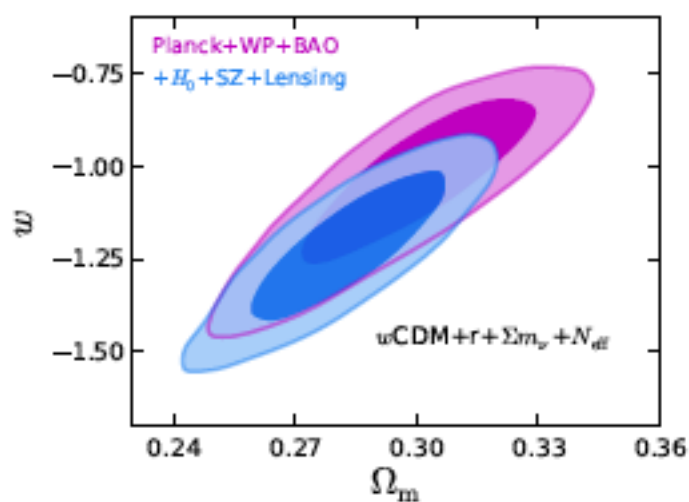
Dark energy and neutrinos

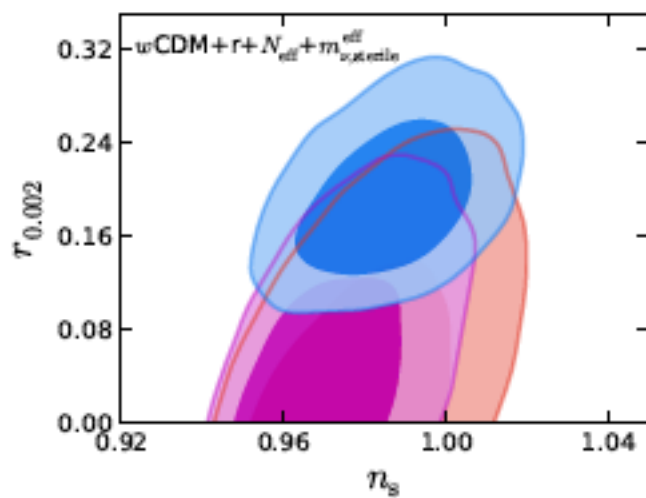
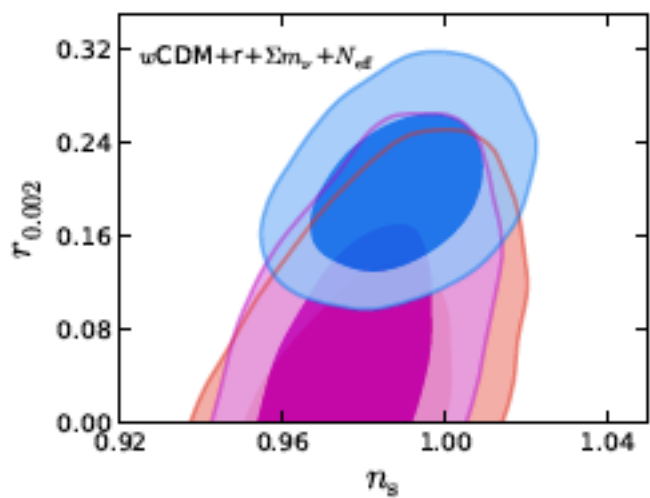
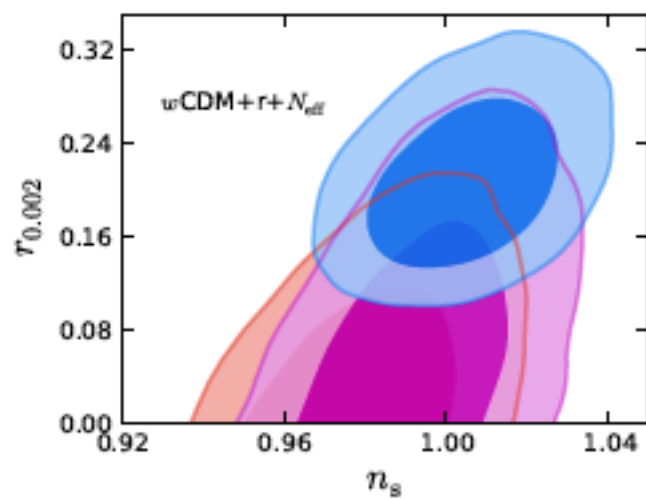
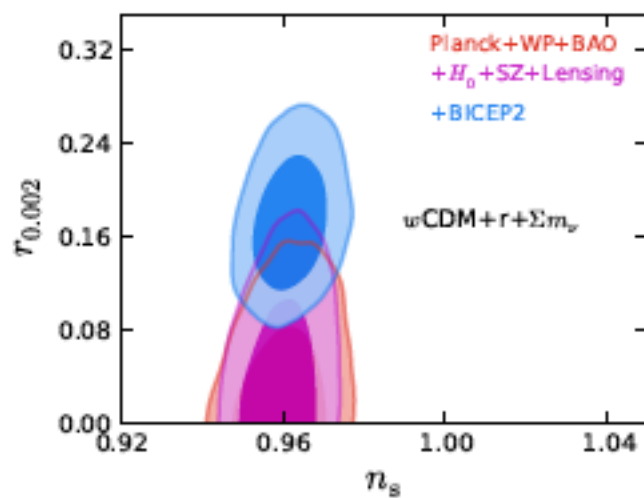


Planck+WP+BAO

Parameter	Λ CDM+ r		Λ CDM+ $r+N_{\text{eff}}+m_{\nu,\text{sterile}}^{\text{eff}}$		w CDM+ r		w CDM+ $r+N_{\text{eff}}+m_{\nu,\text{sterile}}^{\text{eff}}$	
	68% limits	tension	68% limits	tension	68% limits	tension	68% limits	tension
H_0	$67.80^{+0.64}_{-0.63}$	2.4σ	$70.8^{+1.7}_{-2.1}$	1.0σ	$69.0^{+1.8}_{-2.6}$	1.6σ	$71.9^{+2.3}_{-3.2}$	0.6σ
$\sigma_8(\Omega_{\text{m}}/0.27)^{0.3}$	0.857 ± 0.015	4.3σ	$0.842^{+0.038}_{-0.029}$	2.0σ	0.868 ± 0.026	3.2σ	$0.840^{+0.039}_{-0.034}$	1.7σ
$\sigma_8(\Omega_{\text{m}}/0.27)^{0.46}$	$0.876^{+0.019}_{-0.018}$	2.3σ	$0.858^{+0.038}_{-0.030}$	1.7σ	$0.882^{+0.024}_{-0.023}$	2.3σ	$0.853^{+0.041}_{-0.033}$	1.5σ
$r_{0.002}$	< 0.12 (95%)		< 0.20 (95%)		< 0.11 (95%)		< 0.20 (95%)	







Justify extensions of the base standard cosmology

To see how w (DE) improves the fits:

Take corresponding Λ CDM-based models as references

$$\Delta\chi^2 = -10.54 \text{ for the } w\text{CDM}+r+\sum m_\nu \text{ model}$$

$$\Delta\chi^2 = -3.64 \text{ for the } w\text{CDM}+r+N_{\text{eff}} \text{ model}$$

$$\Delta\chi^2 = -4.28 \text{ for the } w\text{CDM}+\bar{r}+\sum m_\nu+N_{\text{eff}} \text{ model}$$

$$\Delta\chi^2 = -7.5 \text{ for the } w\text{CDM}+r+N_{\text{eff}}+m_{\nu,\text{sterile}}^{\text{eff}} \text{ model}$$

To see how neutrinos/dark radiation improves the fits in the framework of w CDM:

Take $w\text{CDM}+r$ model as reference

$$\Delta\chi^2 = -18.12 \text{ for the } w\text{CDM}+r+\sum m_\nu \text{ model}$$

$$\Delta\chi^2 = -1.36 \text{ for the } w\text{CDM}+r+N_{\text{eff}} \text{ model}$$

$$\Delta\chi^2 = -18.42 \text{ for the } w\text{CDM}+\bar{r}+\sum m_\nu+N_{\text{eff}} \text{ model}$$

$$\Delta\chi^2 = -22.02 \text{ for the } w\text{CDM}+r+N_{\text{eff}}+m_{\nu,\text{sterile}}^{\text{eff}} \text{ model}$$

Summary

- BICEP2 indicates that the standard cosmology should at least be extended to the 7-parameter Λ CDM+ r model
- In this model, Planck is still in tension with BICEP2
- Including $dn_s/d\ln k$ can relieve this tension but challenge the design of inflation
- In fact, Planck is also in tensions with other astrophysical observations, such as H_0 , SZ cluster counts, and cosmic shear, etc.
- A sterile neutrino cosmological model could be a concordance model, leading to the consistency of all these data
- Cosmological evidence of the existence of light sterile neutrino is found with high statistical significance
- Model of sterile neutrino is better than that of active neutrino plus dark radiation
- GR test via RSD favors sterile neutrino
- Data further favor extensions to DE+neutrino/DR models

Thanks!